

O największej molekułe mionowej

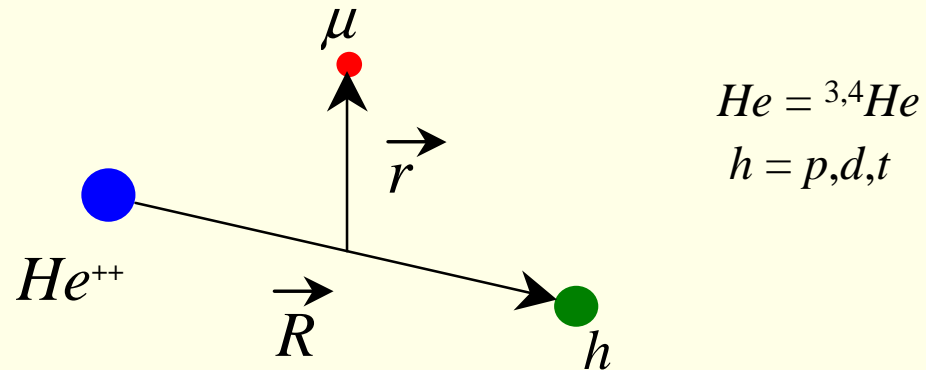


Wilhelm Czapliński AGH WFIS KZPJ ZMJ

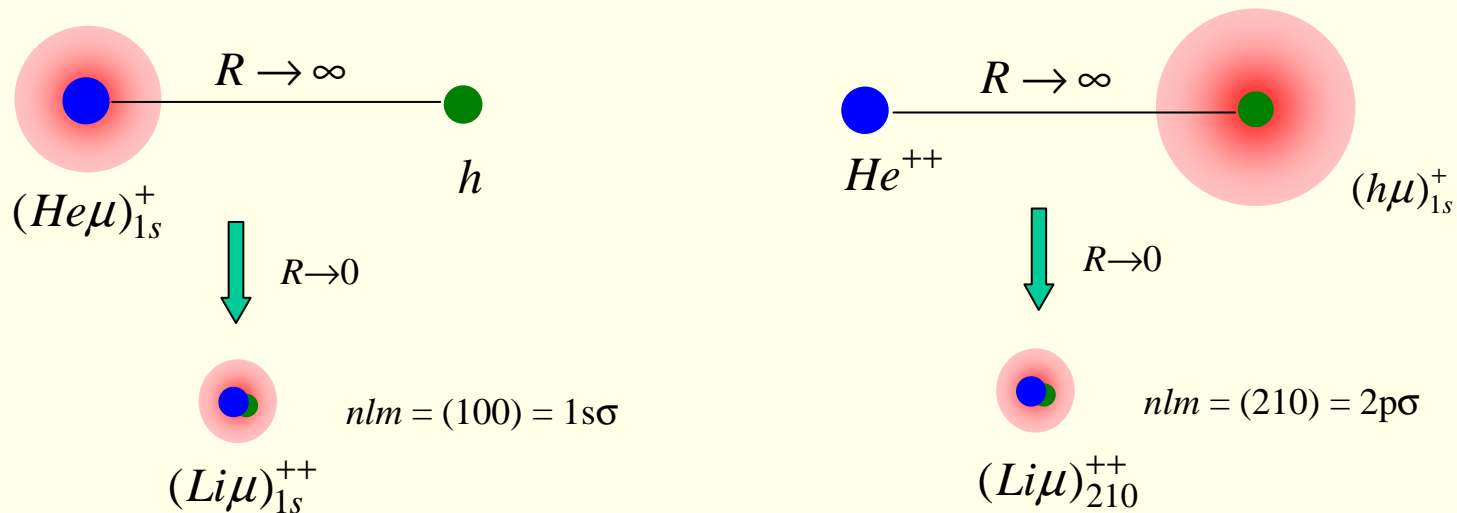
przy współpracy Tomasza Nogi (5.r. FJ)

Rozważania ogólne

Molekuła mionowa wodoru i helu – układ trzech ciał oddziałujących kulombowsko

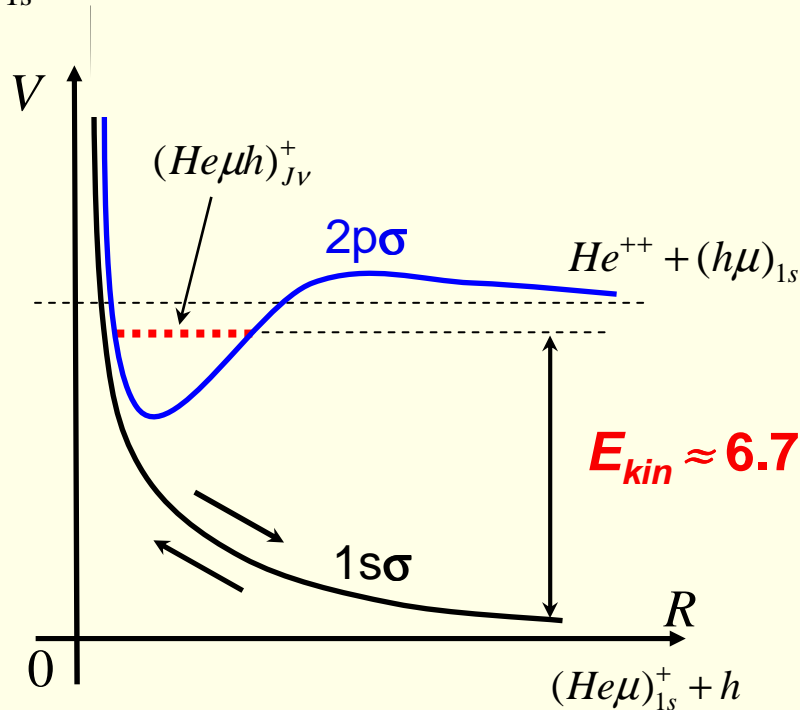
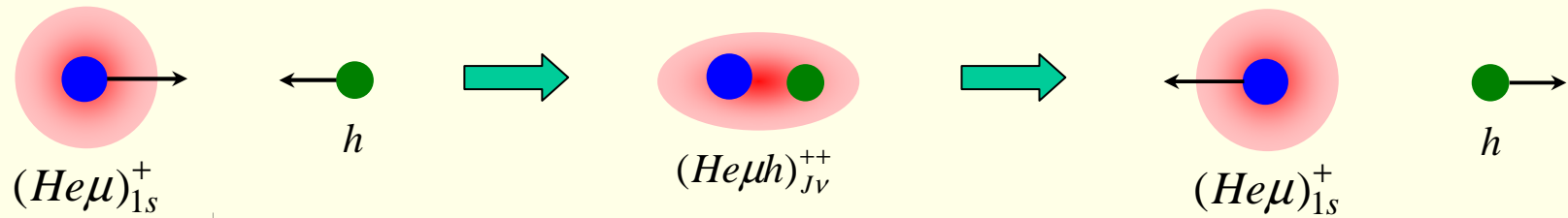


dwa różne stany asymptotyczne ($R \rightarrow \infty$)



Powstawanie molekuly mionowej:

rezonans Feshbacha w zderzeniu w stanie $1s\sigma$

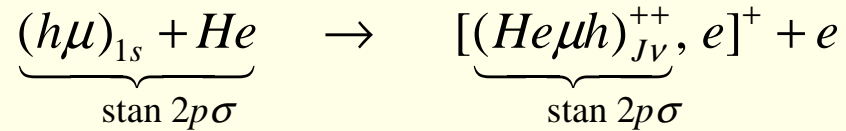


$$V_{2p\sigma}(R) = E_{2p\sigma}^{\mu}(R) + \frac{2}{R} + \frac{J(J+1)}{2MR^2}$$

$$V_{1s\sigma}(R) = E_{1s\sigma}^{\mu}(R) + \frac{2}{R} + \frac{J(J+1)}{2MR^2}$$

Konieczna duza energia zderzenia dla zasiedlenia stanu rezonansowego

Spontaniczne obsadzenie stanu rezonansowego:



$$\Psi_i = \Phi_{2p\sigma}^{\mu}(R, \vec{r}) \phi_i^{rel}(\vec{R}) \phi_i^e(\vec{r}_1, \vec{r}_2) \quad \Psi_f = \Phi_{2p\sigma}^{\mu}(R, \vec{r}) \phi_{J\nu}^{rel}(\vec{R}) \phi_{1s, \vec{k}}^e(\vec{r}_1, \vec{r}_2)$$

energie wiązania (eV)

średnia odlegość między jądrami (μu) $\langle R \rangle_{J\nu} = \langle \phi_{J\nu}^{rel} | R | \phi_{J\nu}^{rel} \rangle$

$1\mu u \approx 1/200 \text{ au}$

(J,ν)	³ Heμp	⁴ Heμp	³ Heμd	⁴ Heμd	³ Heμt	⁴ Heμt
(0,0)	67.2 5.0	73.85 4.89	69.5 4.93	77.5 4.8	71.6 4.89	80.5 4.75
(0,1)						0.076 32.52
(1,0)	34.2 5.49	41.6 5.29	46.5 5.2	55.9 5.02	52.4 5.09	62.9 4.9
(2,0)			97.25 6.2	17.7 5.64	18.2 5.67	30.7 5.3

J.Gronowski, W.Czapliński,
N.Popov,
Acta.Phys.Pol. 106(12004)795

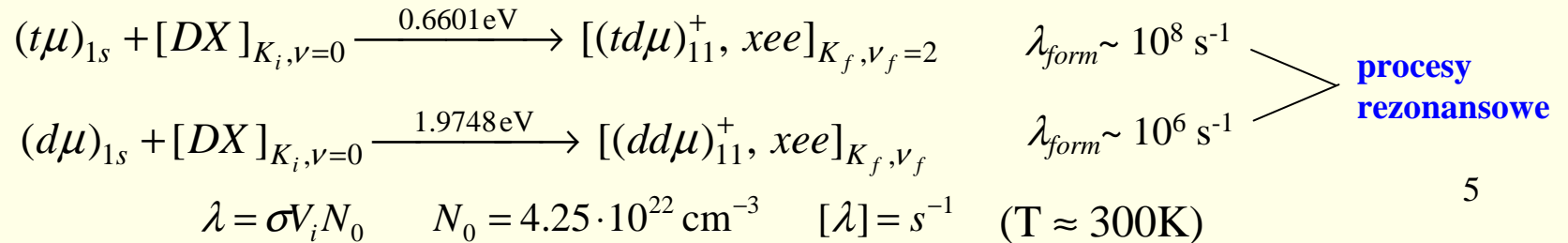
$$\langle R \rangle_{01} = 32.52\mu u \approx 32.52/200 a_B \approx 0.16a_B$$

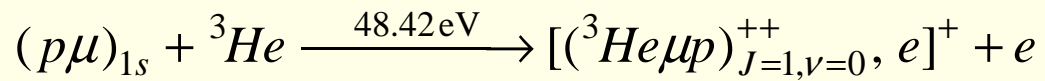
Mionowe molekuly wodoru

A. Scrinzi, H. J. Monkhorst, and S. A. Alexander *Phys.Rev. A38 (1988) 4859*

TABLE II. Expectation values of the interparticle distances and the distances squared of the muonic molecular ions $xy\mu$ ($x, y = p, d, t$). The accuracy is estimated to be ± 2 in the last digit or better. The distances are in absolute muonic units, i.e., $a_\mu = m_e/m_\mu a_0 = 255.92770 \times 10^{-15}$ m. Binding energies (BE) are in eV. $\delta_{xy} = \Delta_{xy} / \langle r_{xy} \rangle$, where $\Delta_{xy} = (\langle r_{xy}^2 \rangle - \langle r_{xy} \rangle^2)^{1/2}$.

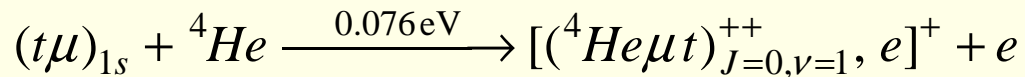
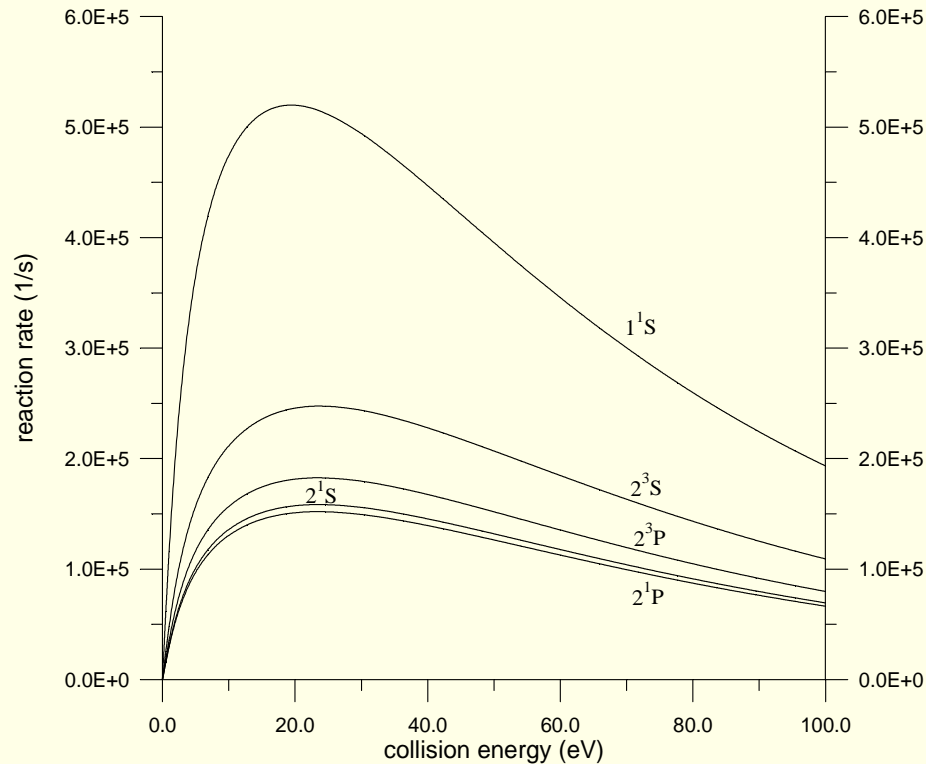
	BE	$\langle r_{x\mu} \rangle$	$\langle r_{y\mu} \rangle$	$\langle r_{xy} \rangle$	$\langle r_{x\mu}^2 \rangle$	$\langle r_{y\mu}^2 \rangle$	$\langle r_{xy}^2 \rangle$	δ_{xy}
$tt\mu(0,0)$	362.9097	2.017 37	20.17 37	2.652 82	5.312 90	5.312 90	7.662 14	0.297 93
$tt\mu(0,1)$	83.7712	2.965 84	2.965 84	4.453 66	12.8861	12.8861	22.4769	0.364 95
$tt\mu(1,0)$	289.1417	2.1231	2.1231	2.8648	5.9059	5.9059	8.9042	0.291 44
$tt\mu(1,1)$	45.2058	3.242	3.242	5.003	15.81	15.81	28.41	0.367 47
$td\mu(0,0)$	319.1397	2.0237	2.1178	2.7479	5.3970	5.8818	8.2872	0.312 25
$td\mu(0,1)$	34.8344	2.738	3.933	5.161	11.75	22.39	30.61	0.386 20
$td\mu(1,0)$	232.4715	2.1567	2.2806	3.0272	6.1890	6.8751	10.0185	0.305 37
$td\mu(1,1)$	0.6601	2.308	8.67	9.428	9.243	128.9	133.0	0.704 47
$dd\mu(0,0)$	325.0735	2.120	2.120	2.834	5.945	5.945	8.876	0.324 25
$dd\mu(0,1)$	35.8443	3.616 30	3.616 30	5.694 74	20.5410	20.5410	37.4188	0.392 21
$dd\mu(1,0)$	226.6816	2.2862	2.2862	3.1668	6.9693	6.9693	11.031	0.316 15
$dd\mu(1,1)$	1.9748	5.416	5.416	9.270	58.5	58.5	112.0	0.550 76
$tp\mu(0,0)$	213.8401	2.002 01	2.461 28	3.036 54	5.4044	8.031	10.347	0.349 52
$tp\mu(1,0)$	99.1265	2.1381	2.9027	3.5846	6.3136	11.3517	14.4619	0.354 25
$dp\mu(0,0)$	221.5494	2.0876	2.4514	3.1007	5.8965	8.0334	10.8291	0.355 25
$dp\mu(1,0)$	97.4981	2.264 80	2.919 93	3.712 64	7.135 61	11.6455	15.5614	0.359 12
$pp\mu(0,0)$	253.1523	2.385	2.385	3.298	7.769	7.769	12.38	0.371 75
$pp\mu(1,0)$	107.2659	2.7790	2.7790	4.0822	10.8982	10.8982	18.9828	0.372 99





proces nierezonansowy

energia wiązania wystarcza do zjonizowania helu (24.6eV)

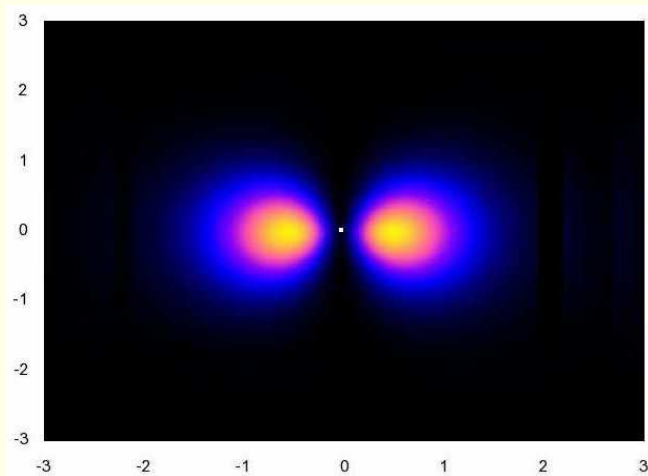


energia wiązania NIE wystarcza do zjonizowania helu (24.6 eV) → konieczna duża energia zderzenia!

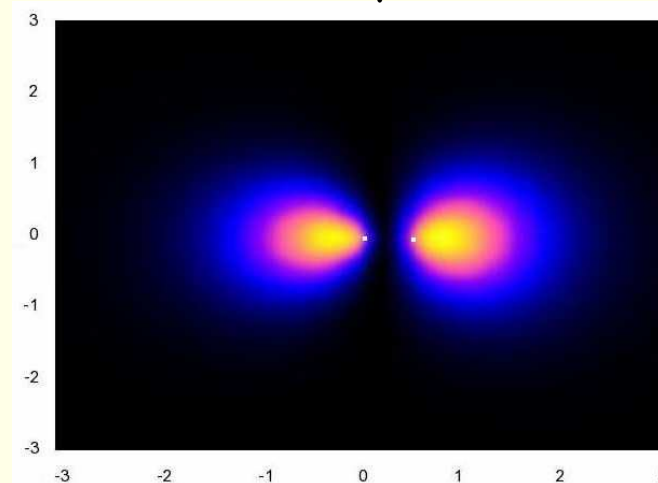
Geometria molekuly $({}^4\text{He}\mu t)_{J=0, \nu=1}^{++}$

Gęstość prawdopodobieństwa $|\Phi_{2p\sigma}^{\mu}(R, \vec{r})|^2$, (He^{++} z lewej strony)

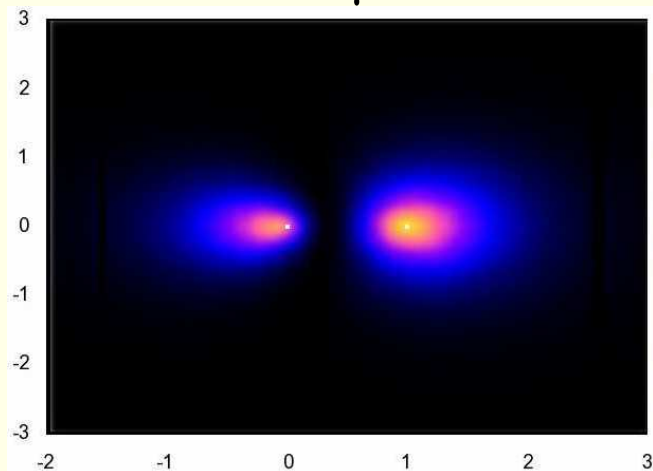
$R = 0.005 \mu\text{u}$



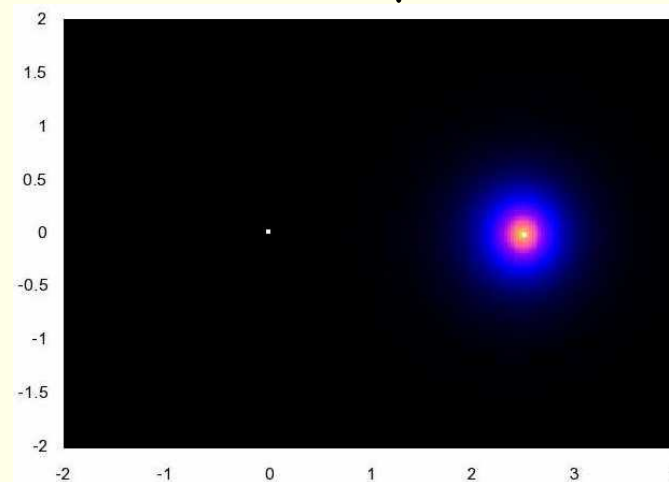
$R = 0.5 \mu\text{u}$



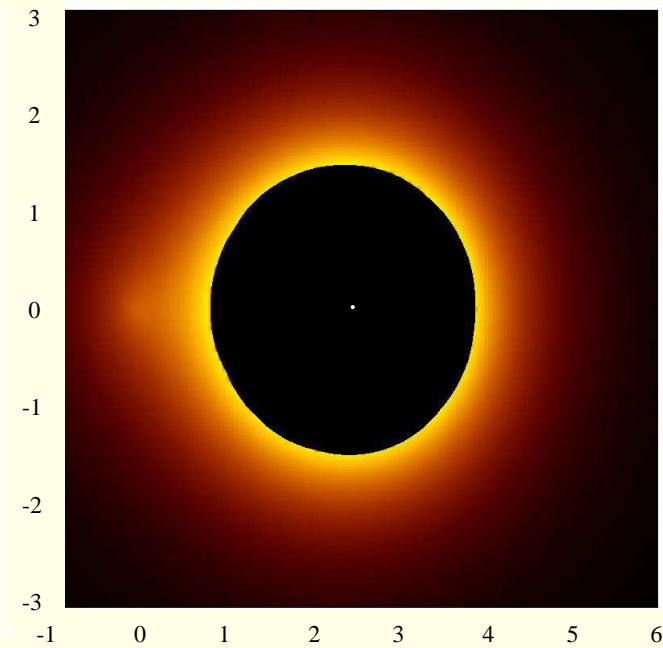
$R = 1.5 \mu\text{u}$



$R = 2.5 \mu\text{u}$

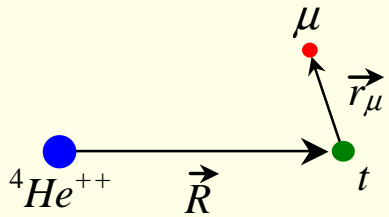


$$R = 2.5 \mu u$$

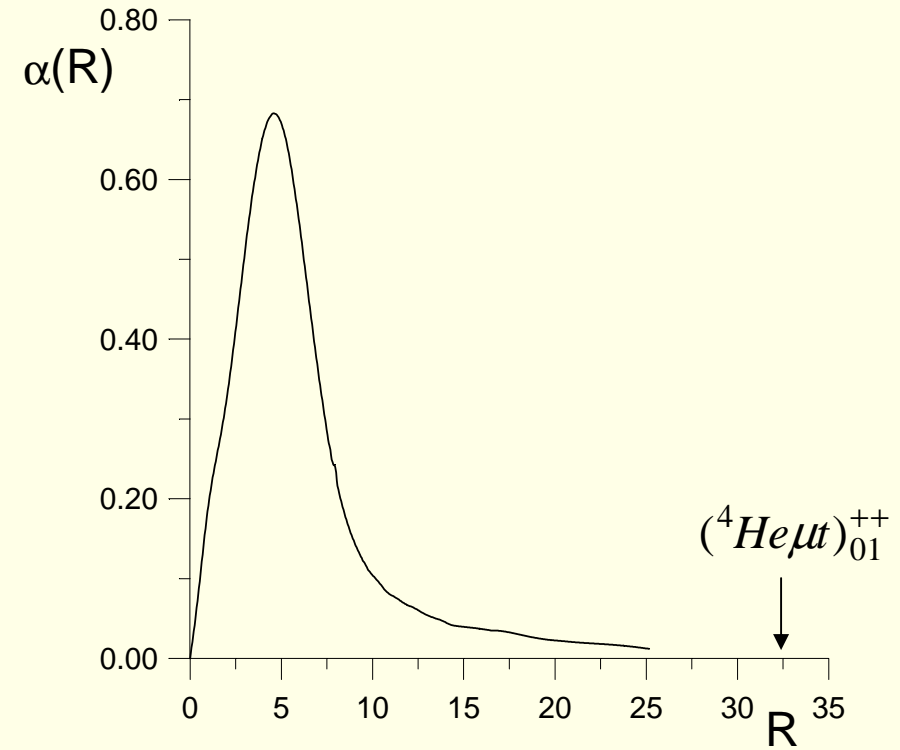


$$\Phi_{2p\sigma}^{\mu}(R, \vec{r}) \xrightarrow{R \rightarrow \infty} \Phi_{1s}^{t\mu}(\vec{r}) = \frac{e^{-r}}{\sqrt{\pi}}$$

$$E_{2p\sigma}^{\mu}(R) \xrightarrow{R \rightarrow \infty} -0.5$$



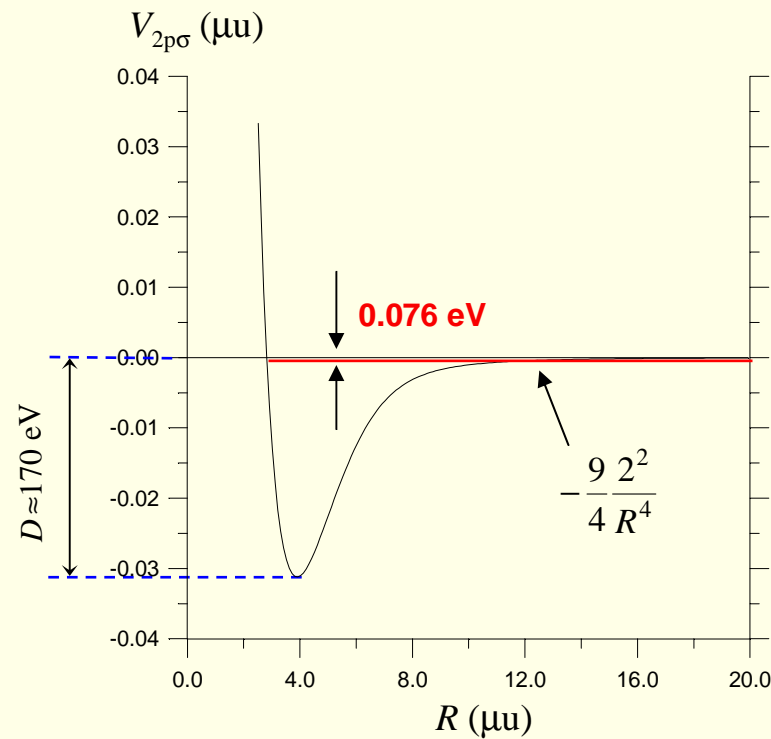
$$\alpha(R) = -\langle \Phi_{2p\sigma}^{\mu} | \vec{r}_{\mu} \cdot \hat{R} | \Phi_{2p\sigma}^{\mu} \rangle \xrightarrow{R \rightarrow \infty} 0$$



WNIOSEK: molekula $({}^4\text{He}\mu t)_{J=0, \nu=1}^{++}$ to układ $(t\mu)_{1s} - {}^4\text{He}^{++}$

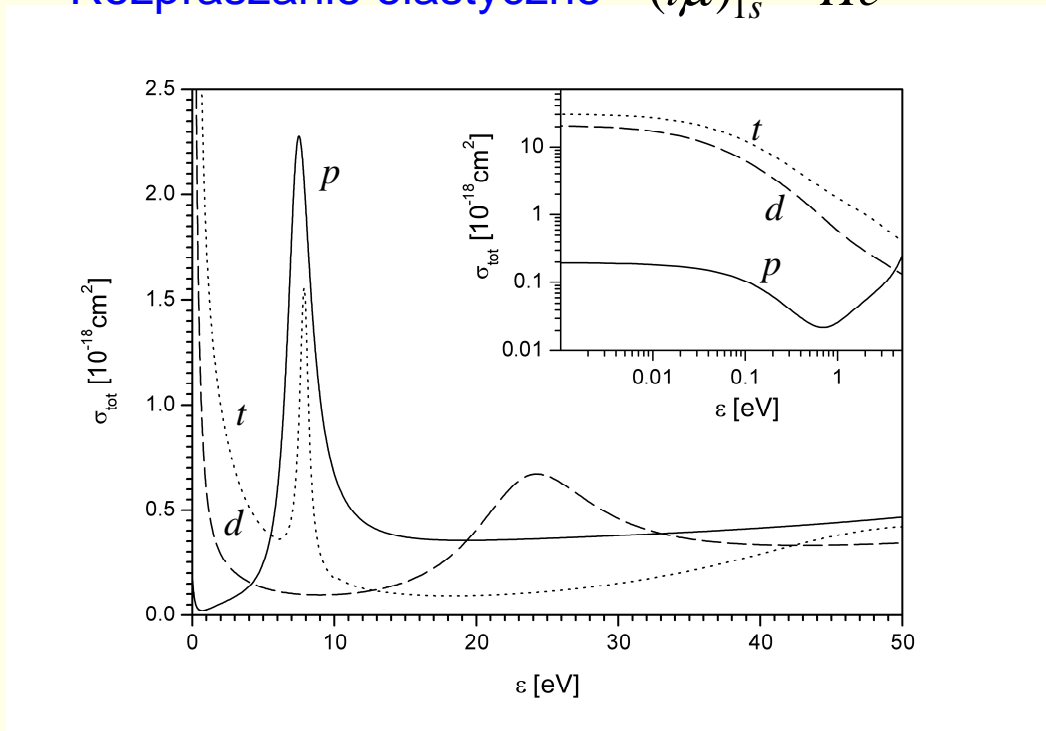
$$V_{2p\sigma}(R) = E_{2p\sigma}^{\mu}(R) - E_{2p\sigma}^{\mu}(\infty) + 2/R \xrightarrow{R \rightarrow \infty} -\frac{9}{4} \frac{2^2}{R^4} + O(R^{-5})$$

kwadratowy efekt Starka



stan słabo-związany „widać” w rozpraszaniu elastycznym

Rozpraszanie elastyczne $(t\mu)_{1s} - {}^4\text{He}^{++}$



$$\alpha_t = + 59.1$$

$$\alpha_d = - 48.0$$

$$\alpha_p = - 4.4$$

$$\sigma_{el} = 4\pi \sum_{J=0}^{\infty} (2J+1) \frac{\sin^2(\delta_J)}{k^2} \xrightarrow{k \ll 1} 4\pi \frac{\sin^2(\delta_0)}{k^2} \xrightarrow{k \rightarrow 0} 4\pi\alpha^2 = \frac{4\pi}{2M E_{01}^b}$$

$$\alpha = - \lim_{k \rightarrow 0} \frac{\sin(\delta_0)}{k} \text{ - długość rozpraszania}$$

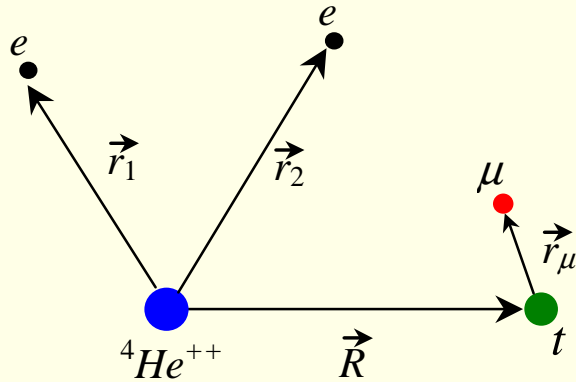
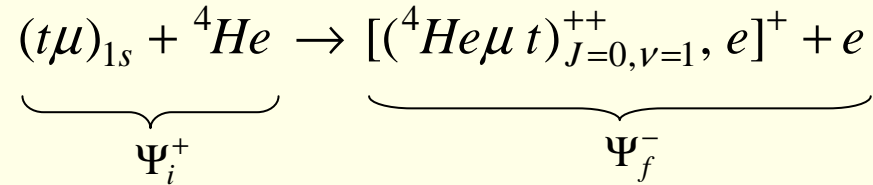
energia wiązania

$\alpha \ll -1$ stan wirtualny (anty-związany)

$\alpha \gg 1$ stan słabo związany \rightarrow **molekuła** $({}^4\text{He}\mu t)_{01}^{++}$

$$E_{01}^b = \frac{4\pi}{2M \sigma_{el}} \approx 0.074 \text{ eV}$$

Obliczenie przekroju czynnego na powstawanie molekuly w procesie



Przybliżenie adiabatyczne:

$$\Psi_i^+ = \Phi_{2p\sigma}^\mu(R, \vec{r}) \phi_i^{rel+}(\vec{R}) \phi_{1^1s}^e(\vec{r}_1, \vec{r}_2)$$

$$\Psi_f^- = \Phi_{2p\sigma}^\mu(R, \vec{r}) \phi_{01}^{rel}(\vec{R}) \phi_{1s, \vec{k}}^{e-}(\vec{r}_1, \vec{r}_2)$$

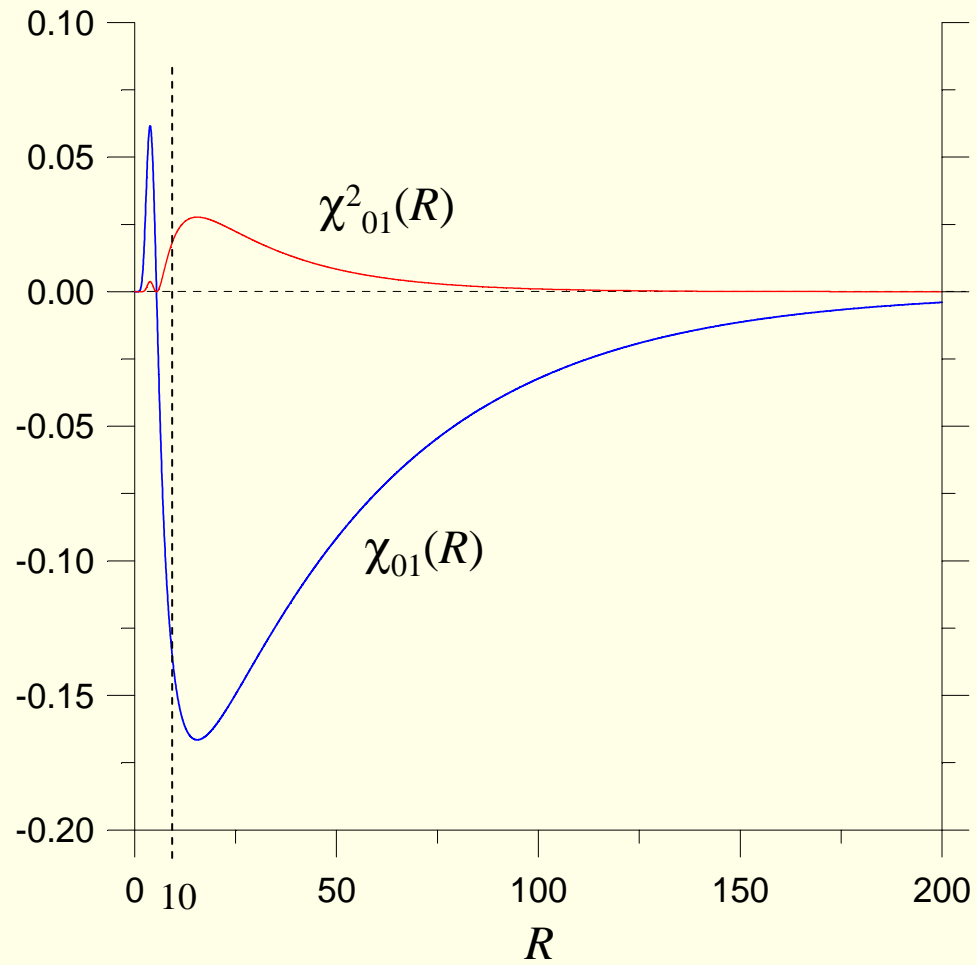
$$\phi_{1^1s}^e(\vec{r}_1, \vec{r}_2) = u(r_1)u(r_2) \quad u(r) = (4\pi)^{-1/2} (Ae^{-ar} + Be^{-br})$$

$$\phi_{1s, \vec{k}}^{e-}(\vec{r}_1, \vec{r}_2) = 2^{-1/2} [R_{1s}(r_1)F_C(\vec{k}_e, \vec{r}_2) + R_{1s}(r_2)F_C(\vec{k}_e, \vec{r}_1)]$$

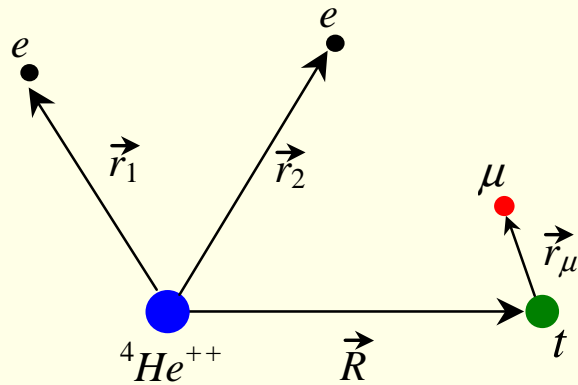
$$\phi_i^{rel+}(\vec{R}) = \frac{1}{k_i R} \sum_{L=0}^{\infty} i^L (2L+1) e^{i\delta_L} \chi_{k_i L}(R) P_L(\vec{k}_i \cdot \hat{R})$$

$$\phi_{01}^{rel}(\vec{R}) = \frac{1}{\sqrt{4\pi}} \frac{\chi_{01}(R)}{R}$$

funkcja $\chi_{01}(R)$



oddziaływanie odpowiedzialne za przejście



$$V_{tr} = V_{t\mu-e} = \sum_{i=1}^2 \left(\left| \vec{R} + \vec{r}_\mu - \vec{r}_i \right|^{-1} - \left| \vec{R} - \vec{r}_i \right|^{-1} \right) = \left| \begin{array}{l} r_\mu \ll R \\ r_\mu \ll r_i \end{array} \right| \approx \sum_{i=1}^2 \vec{r}_\mu \cdot \nabla_R \frac{1}{|\vec{R} - \vec{r}_i|}$$

Złota reguła Fermiego

$$d\sigma = \frac{2\pi}{V_i} \left| \langle \Psi_f^- | V_{tr} | \Psi_i^+ \rangle \right|^2 \delta(E_i - E_f) \frac{d^3 k_e}{(2\pi)^3}$$

$$\langle \Phi_{2p\sigma}^\mu | V_{tr} | \Phi_{2p\sigma}^\mu \rangle = \underbrace{\langle \Phi_{2p\sigma}^\mu | \vec{r}_\mu \cdot \hat{R} | \Phi_{2p\sigma}^\mu \rangle}_{-\alpha(R)} \sum_{i=1}^2 \frac{\partial}{\partial R} \frac{1}{|\vec{R} - \vec{r}_i|} = -\alpha(R) \sum_{i=1}^2 \sum_{l=0}^{\infty} a_l(R, r_i) P_l(\hat{R} \cdot \hat{r}_i)$$

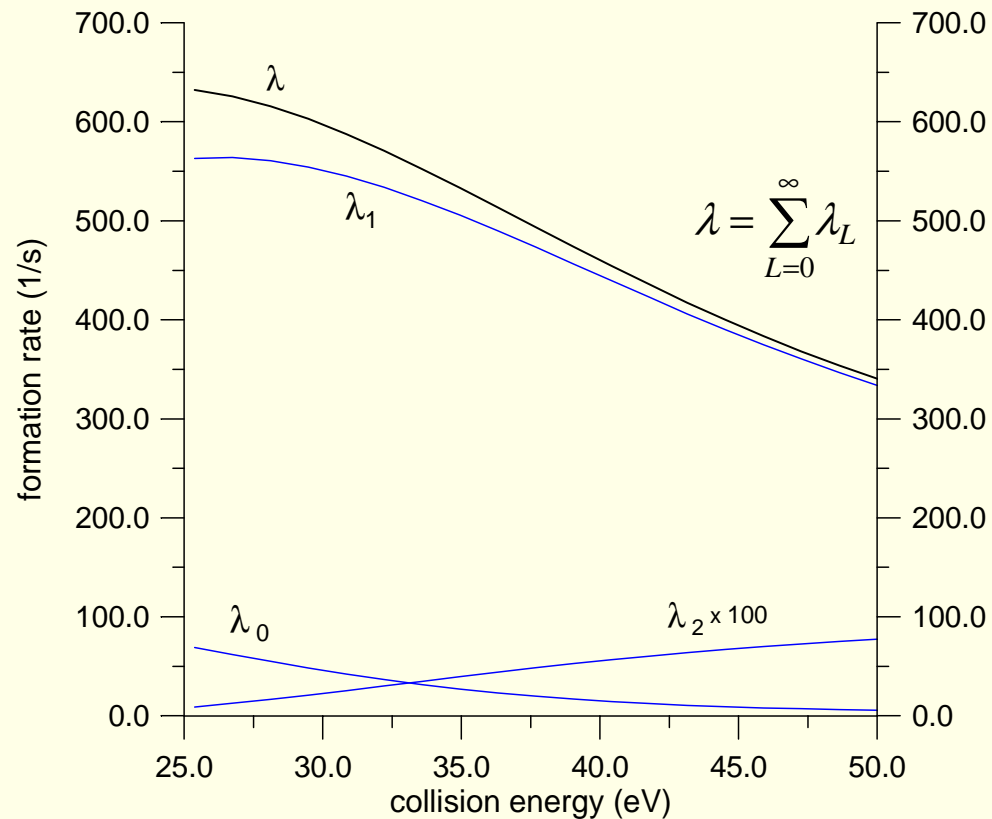
$$a_l(R, r) = \frac{1}{R^2} \begin{cases} -(l+1)(r/R)^l & r < R \\ l(R/r)^{l+1} & r > R \end{cases}$$

$$\sigma = \frac{2^{10} \pi}{V_i k_i^2 k_e} \sum_{L=0}^{\infty} \frac{K_L^2}{2L+1} \quad K_L = \int_0^{\infty} \chi_{k_i L}(R) \chi_{01}(R) \alpha(R) Q_L^e(R) R^{-2} dR$$

↙ czynniki elektronowy

Szybkość powstawania molekuly reakcji (formation rate)

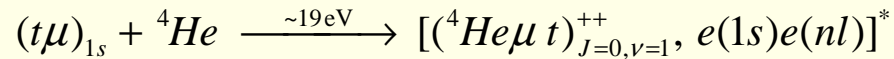
$$\lambda = \sigma V_i N_0 \quad N_0 = 4.25 \cdot 10^{22} \text{ cm}^{-3} \quad [\lambda] = \text{s}^{-1}$$



WNIOSEK: proces zachodzi bardzo rzadko

Inne procesy powstawania molekuly $({}^4\text{He}\mu t)_{J=0, \nu=1}^{++}$

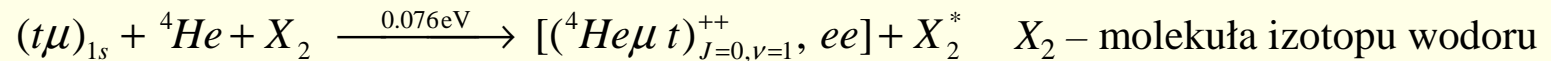
1. Rezonansowy:



- większy czynnik elektronowy $Q_L^e(R)$

- mniejsza energia zderzenia $(t\mu)_{1s} - {}^4\text{He}^{++} \rightarrow$ mniejsze oscylacje f-cji $\chi_{k_i L}(R)$

2. Kwazi-rezonansowy:



- największy czynnik elektronowy $Q_L^e(R)$

- zerowa energia zderzenia

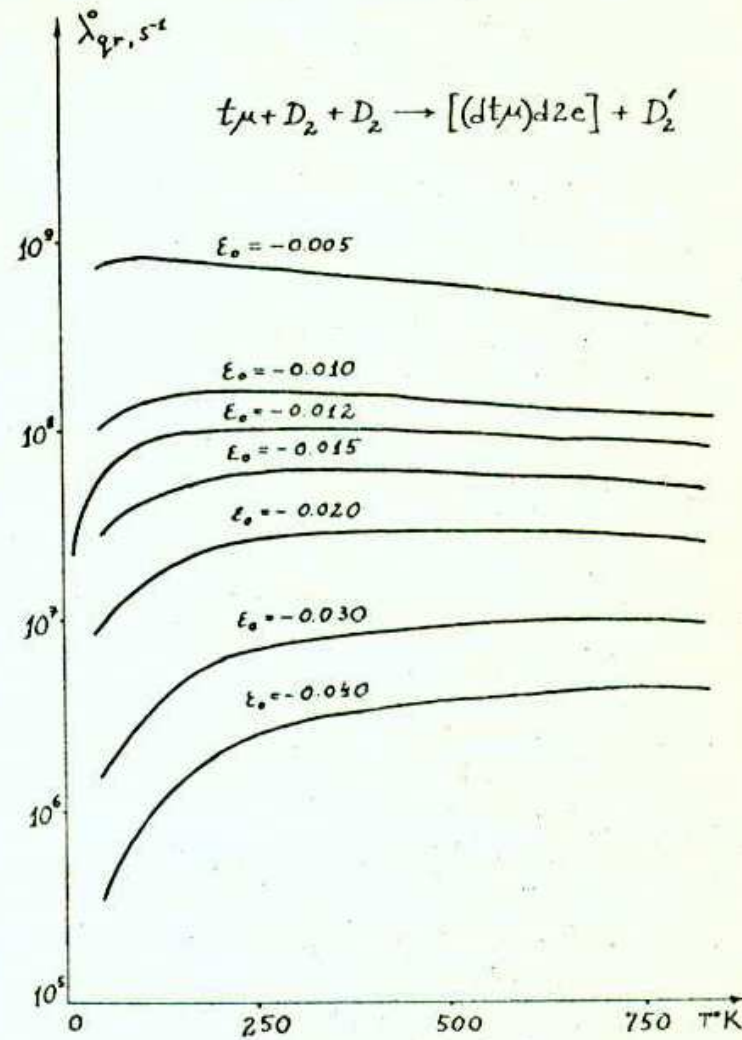
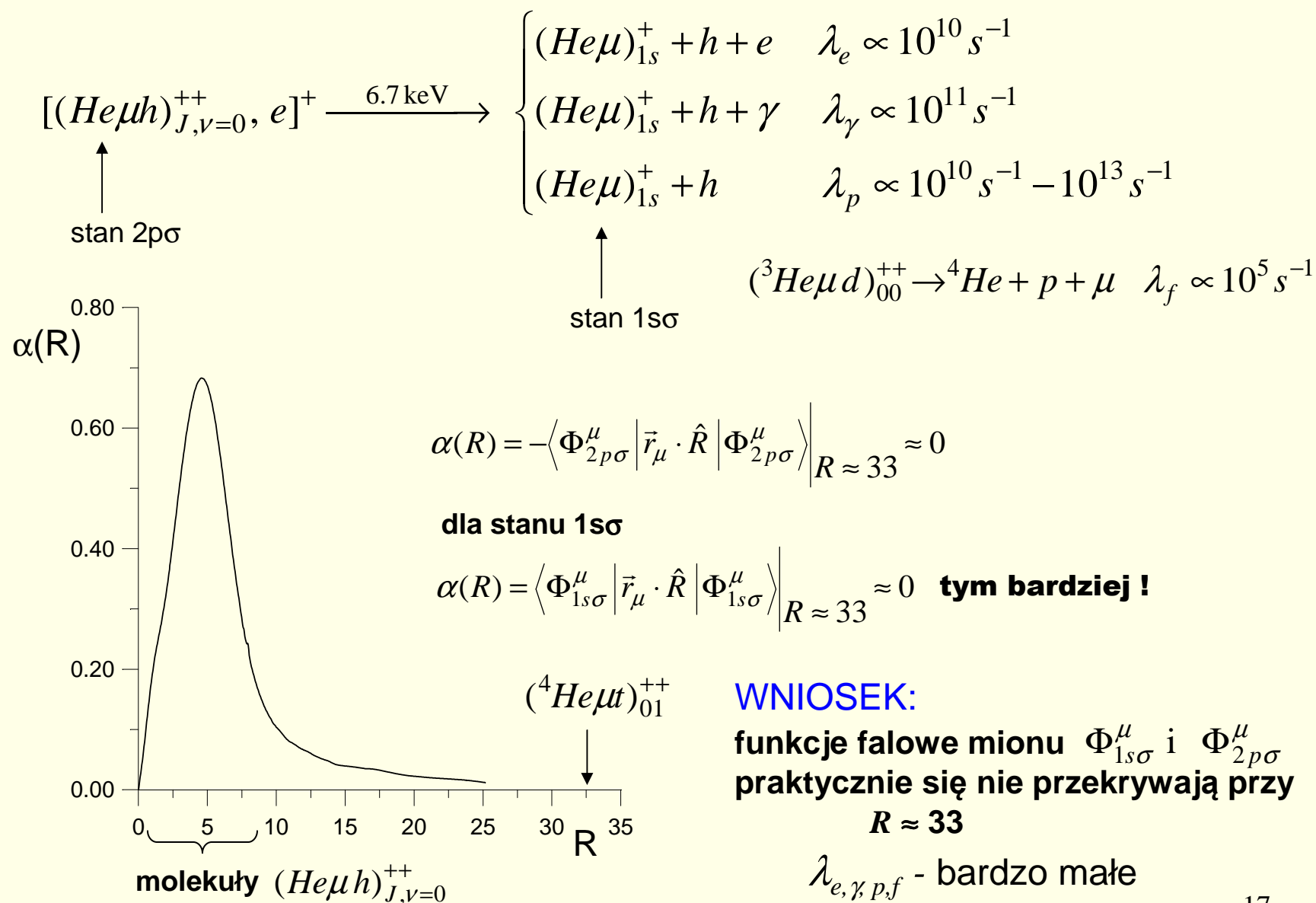


Fig. 3. The rates $\lambda_{qr}^0(\epsilon_0, T)$ of quasiresonant mesic molecule formation (2) at different ϵ_0 normalized to the liquid hydrogen density $\varphi = 1$.

Kanały rozpadu molekuł wodoru i helu



Czy istnieje szybszy rozpad (?)

przejście typu E1

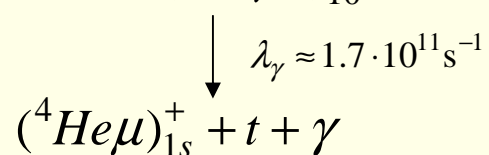
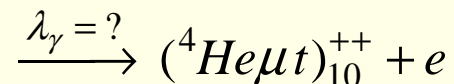
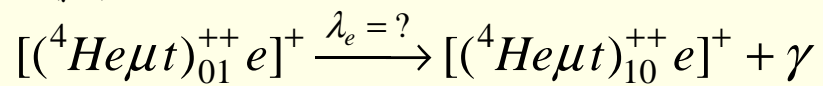
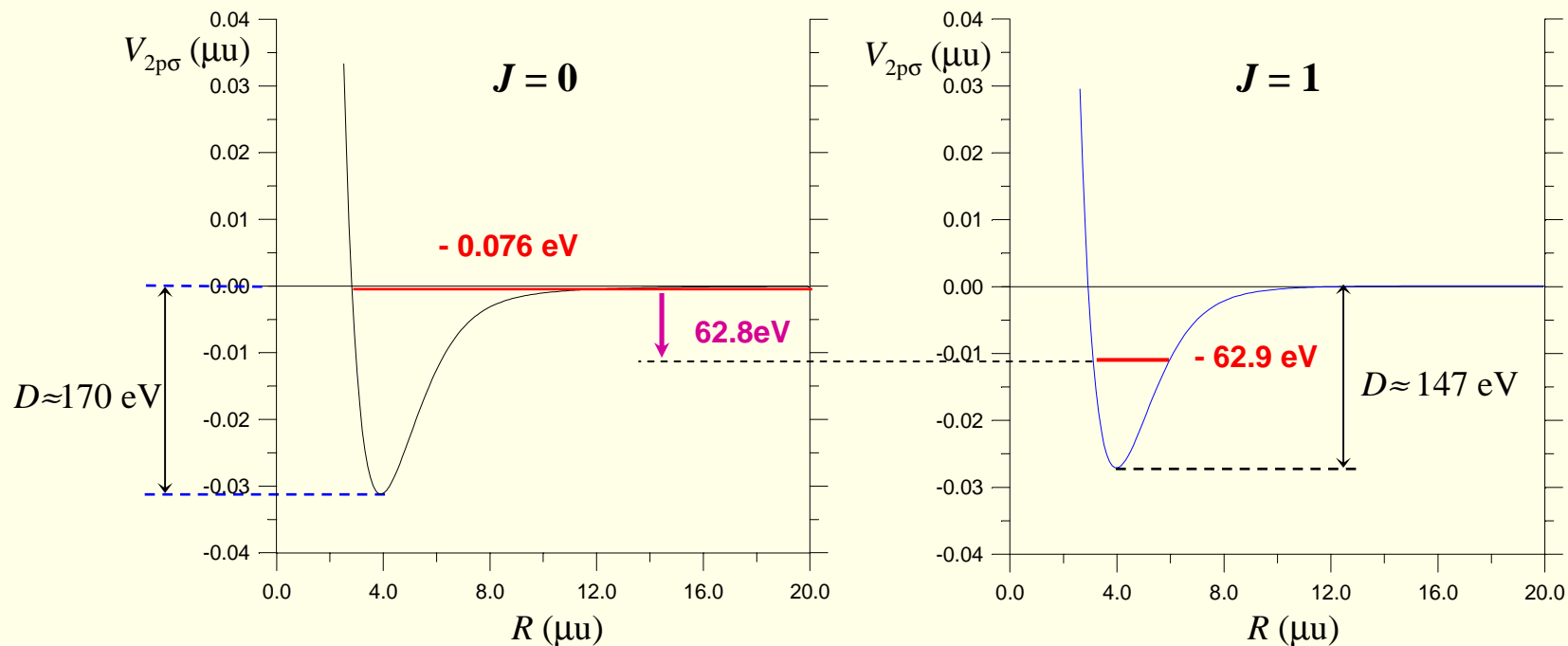
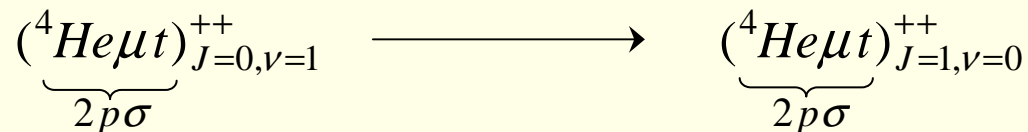


Table 1
 Nonrelativistic energy levels $E_{\nu,J}$ of muonic helium hydride
 below the $d\mu(1s)$ and $t\mu(1s)$ thresholds, given in units of eV.

(ν, J)	${}^3\text{He}d\mu$	$\alpha d\mu$	${}^3\text{He}t\mu$	$\alpha t\mu$
0, 0	-70.976	-79.340	-72.296	-81.335
0, 1	-48.419	-58.222	-53.330	-63.958
0, 2	-9.434	-20.416	-19.379	-32.063
1, 0				-0.134

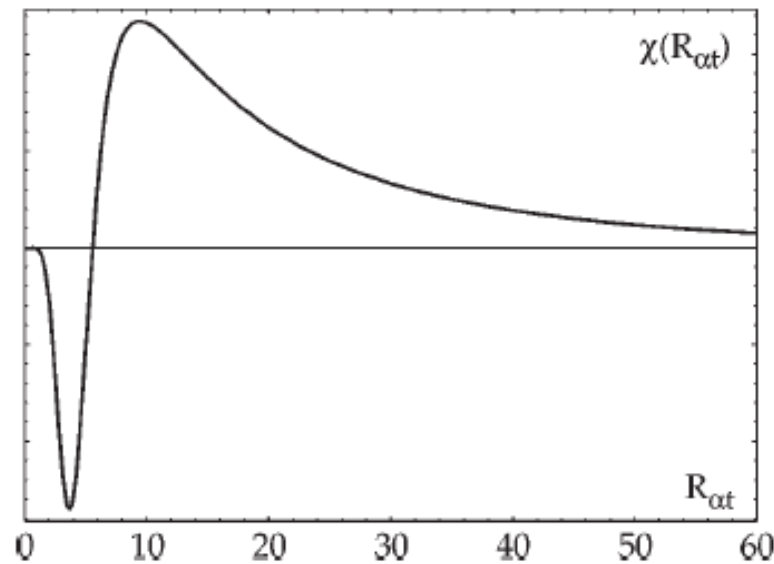


Figure 1. The pseudo-wavefunction $\chi(R_{\alpha t})$ for the $(\nu, J) = (1, 0)$ $\alpha t\mu$ state, with $R_{\alpha t}$ given in units of $m_{\mu} = e^2 = \hbar = 1$.

Dziękuję za uwagę