Transition to Cooperative Behaviour in a Route Choice Game

Martin Schönhof[1,2], Dirk Helbing[1], Hans-Ulrich Stark[1],
and Janusz Hołyst[2]

1) Institute for Transport & Economics, Dresden University of Technology
2) Faculty of Physics, Warsaw University of Technology

Faculty of Physics and Center of Excellence for
Complex Systems Research (CSR),
Warsaw University of Technology

Overview

- Traffic networks, route choice, and game theory
- Route choice experiment (theory and setup)
- Results and cooperative behaviour
- Simulation

Traffic Distribution

- Road network as set of different origin destination pairs (o-d pairs)
- Travel time on a route increases with its occupation
- Wardrop equilibirium (special Nash Equilibrium): for all o-d pairs
  - equal costs (travel times) on all used routes and
  - higher costs on all unused routes
- Wardrop equilibrium mostly not efficient ($\neq$ system optimum), sometimes even suboptimal for all users (Braess' paradox)

Occupation and Inverse Travel Times

- Greenshield's linear velocity-density relation for route $i \in \{A, B\}$:

$$V_i(N_i) = V_i^0 \left(1 - \frac{N_i(t)}{N_i^{\max}}\right)$$

V_i: Average vehicle speed
V_i^0: Maximal velocity
N_i: Number of veh. on route i
$N_i^{\max}$: Capacity of route i

- Inverse travel time $P_i(N_i) = \frac{V_i(N_i)}{S_i} = P_i^0 - P_i^1 N_i$

S_i : Length of route i, $P_i^0 = \frac{V_i^0}{S_i}$, and $P_i^1 = \frac{V_i^0}{N_i^{\max} S_i}$

$$1/T(N_i) = A_i - B_i N_i, \quad \text{inverse travel time}$$

The user equilibrium of equal travel times is found for a fraction

$$\frac{N_1^{\mathrm{e}}}{N} = \frac{B_2}{B_1 + B_2} + \frac{1}{N}\frac{A_1 - A_2}{B_1 + B_2} \tag{3}$$

of persons choosing route 1. In contrast, the system optimum corresponds to the maximum of the overall inverse travel times $N_1/T_1(N_1) + N_2/T_2(N_2)$ and is found for the fraction

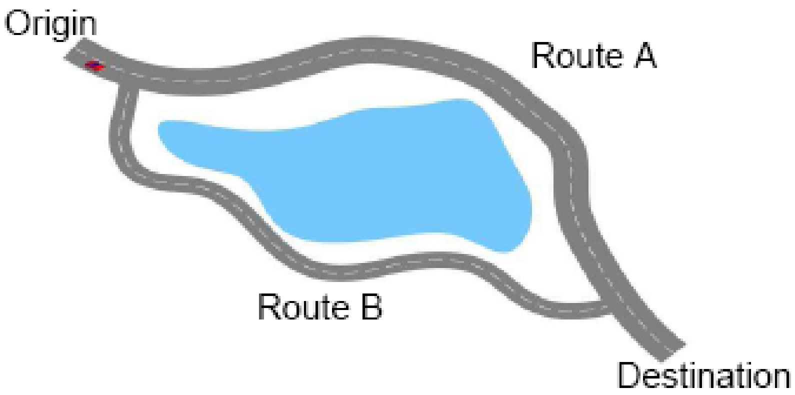
$$\frac{N_1^{\mathrm{o}}}{N} = \frac{B_2}{B_1 + B_2} + \frac{1}{2N}\frac{A_1 - A_2}{B_1 + B_2} \tag{4}$$

If $B_1=B_2=B$ then:

for user optimum: $N_1 - N_2=(A_1 - A_2)/B$ (more users at a faster road)

for system optimum: $N_1-N_2=(A_1-A_2)/(2B)$ (faster road should NOT be overloaded !!!)

Decision Game Derived by a Small Traffic System



- 1 o-d pair
- 2 routes (A, B)
- 2 users
- payoff points (P_A, P_B)
- users on A,B: N_A, N_B

$$P_A(N_A) = 600 - 300N_A$$
$$P_B(N_B) = \quad 0 - 100N_B$$

2 users:

N_A	0	1	2
N_B	2	1	0
P_A	–	300	0
P_B	-200	-100	–
$\overline{P}$	-200	100	0

Game theory – the Route Choice Game

N_A	0	1	2
N_B	2	1	0
P_A	–	300	0
P_B	-200	-100	–
$\overline{P}$	-200	100	0

$\Downarrow$

	Route A	Route B
Route A	0 0	−100 300
Route B	300 −100	−200 −200

- symmetrical 2x2 game
- “A” dominant strategy
- (A,A) is unique Nash-equilibirum which is
  - not system optimal
  - pareto efficient
- (A,A) best choice in one shot game

prisoner's dilemma:

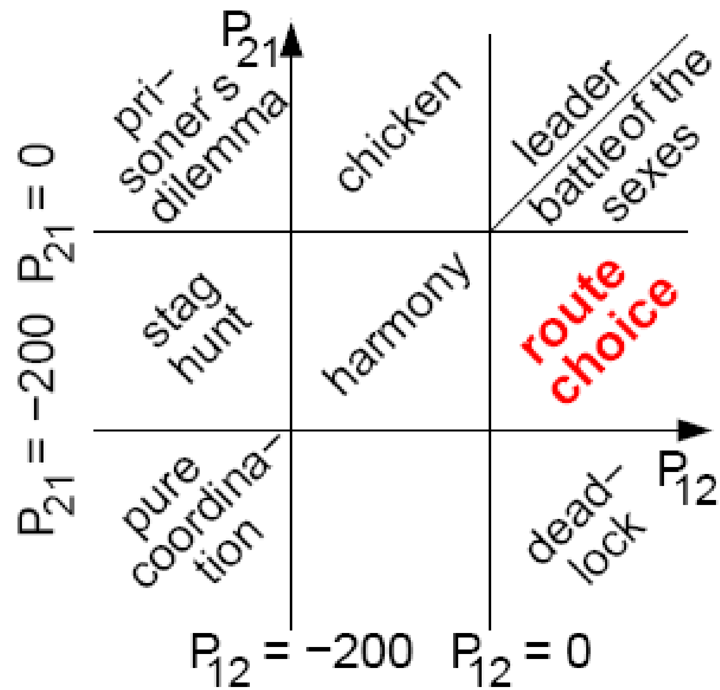
	coop.	def.
coop.	0	−300
def.	100	−200

general form of symmetrical 2x2 games:

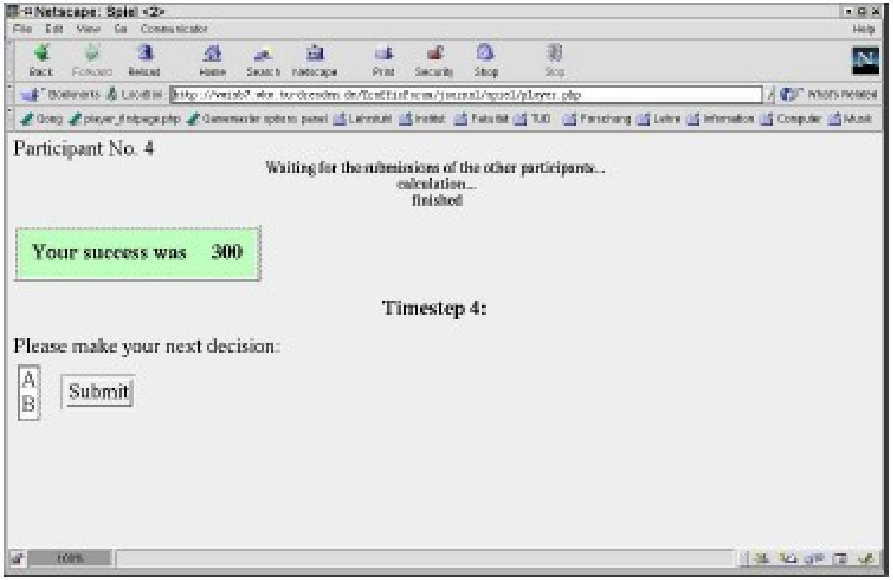
	1	2
strategy 1	0	P_{12}
strategy 2	P_{21}	−200

route choice:

	A	B
route A	0	300
route B	−100	−200



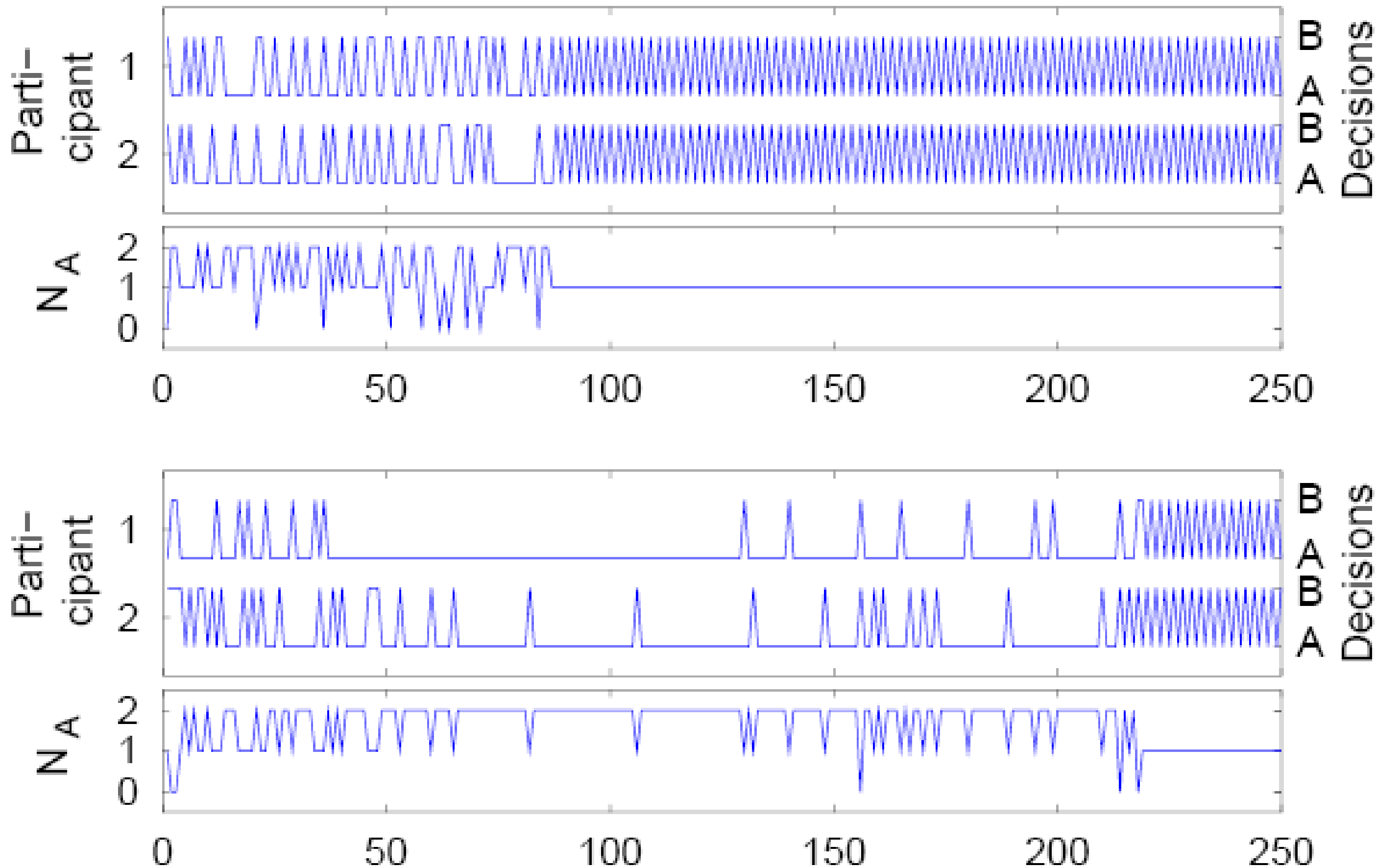
Setup: Screenshot



Player knowledge:

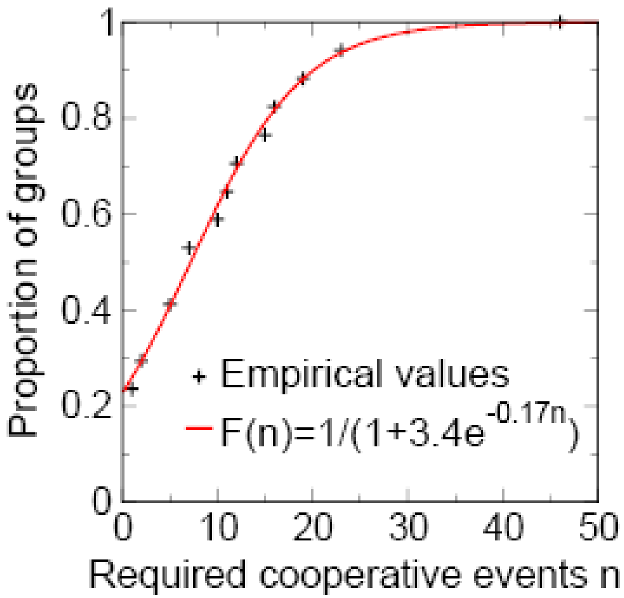
- (A,A)$\Rightarrow$ $P_A = 0$
- (A,B)$\Rightarrow$ $\overline{P} = 100, \quad P_A > P_B$
- time dependent strategy may help to reach $\overline{P} = 100$

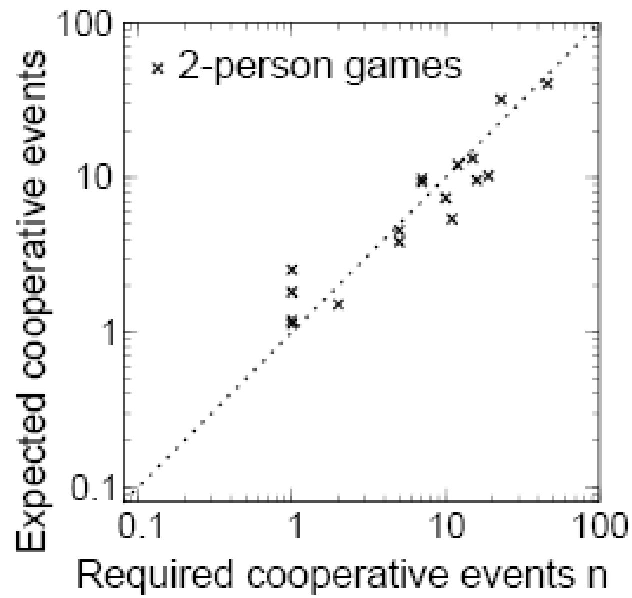
Emergence of Coherent Oscillatory Behaviour



Cooperative Events Before Start of Cooperation

Cooperative event (ce): The participants established the system optimum in step t, and both participants change the route at $t+1$.

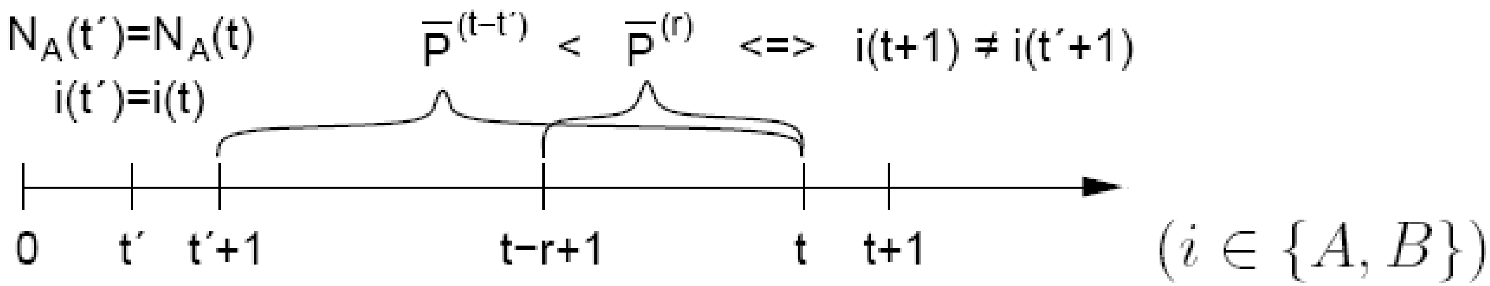




$$n_{\text{expect.}} = \frac{T_{\text{coop}}}{\frac{1}{\text{ce-rate}}} = \frac{T_{\text{coop}}}{2\prod_{i=1}^{2} \frac{1}{\text{changing rate of user } i \text{ (until coop.)}}}$$

Model of Reinforcement Learning

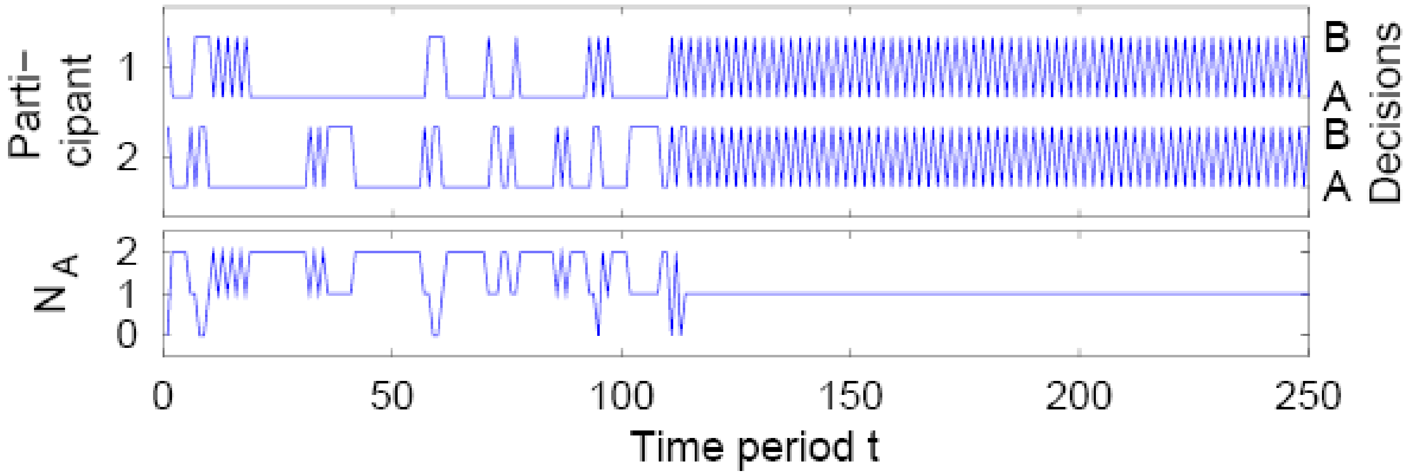
- deterministic preferences of decision behaviour:



- In addition, random change of decision behaviour (mutation):

$$\nu_l(t) = \nu_l^0 + \nu_l^1[1 - \overline{P}_l^{(r)}(t)/100]$$

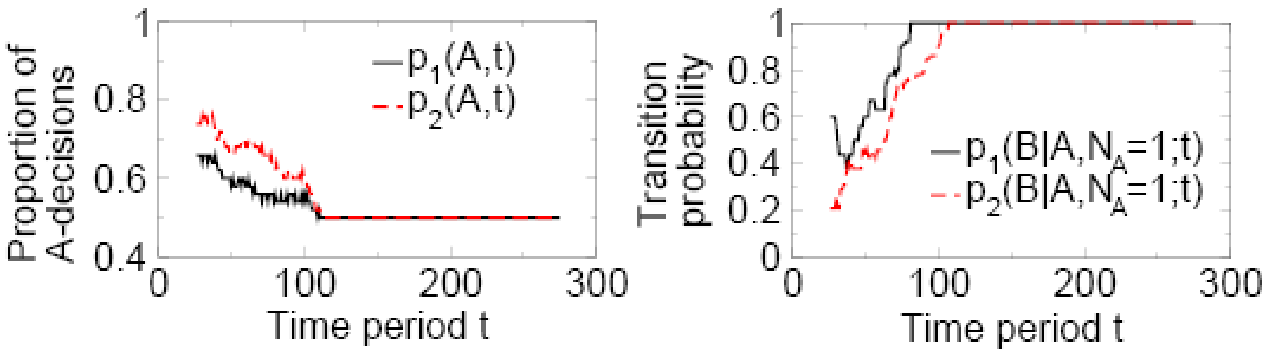
Simulated Route Choice Game



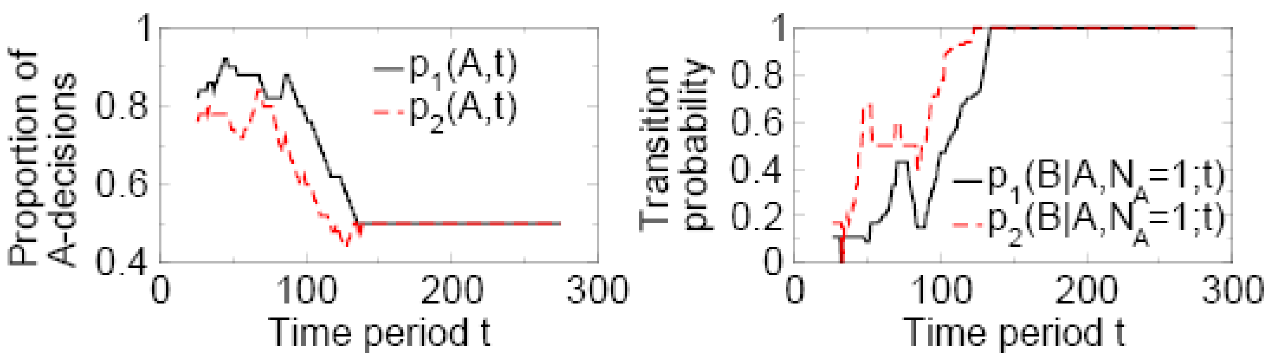
- $r = 2$
- mutation probability: $\nu_l(t) = 0.03[1 - \overline{P}_l^{(2)}(t)/100]$
- Initial conditions:
  - $p_l(B|A, N_A; 0) = 0$ and
  - $p_l(A|B, N_A; 0) = 1$

Comparison of Observables

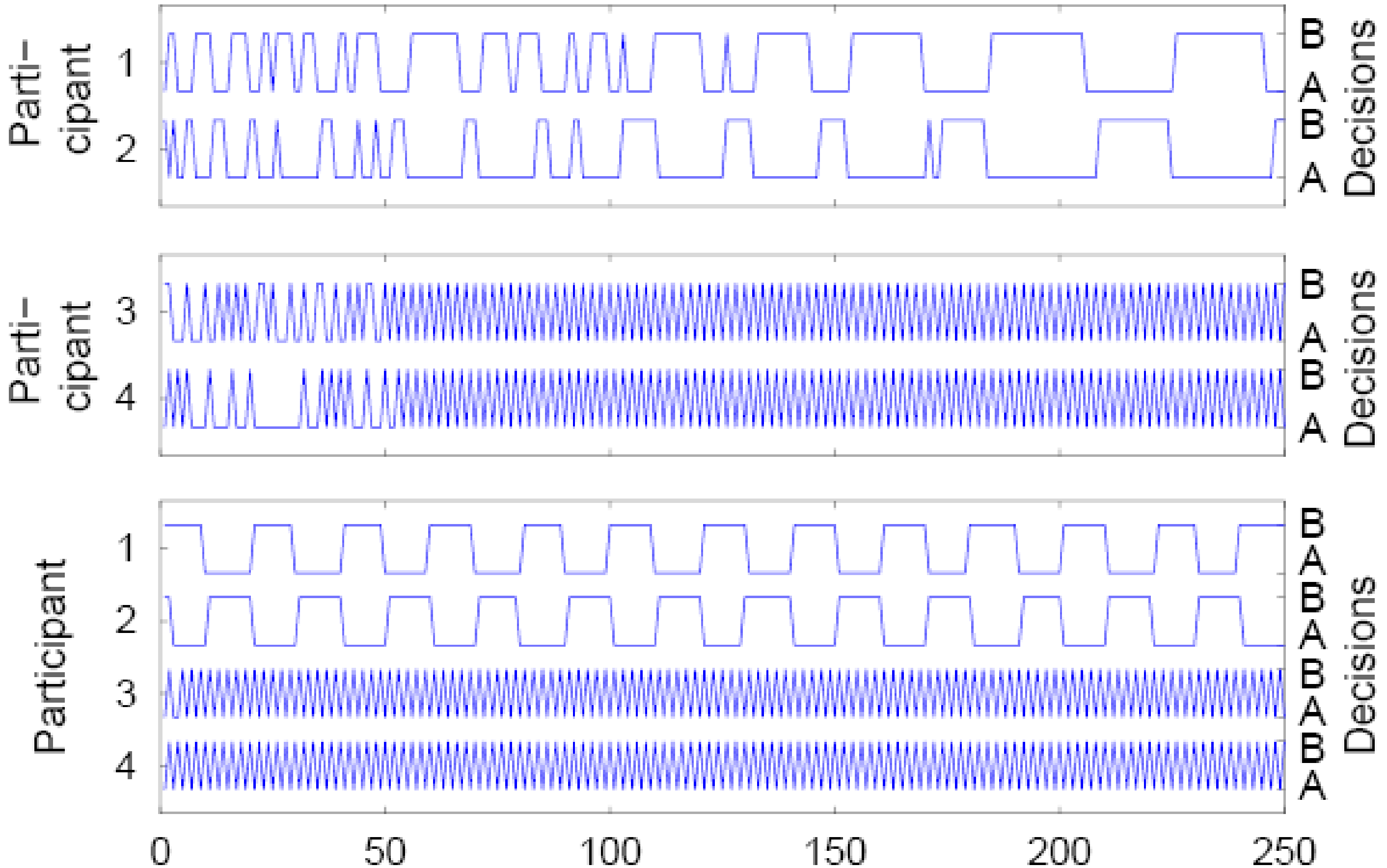
Experiment:



Simulation:



Cooperation in 4 Person Game Enhanced by Experience



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Entscheidungstheoretische Experimente

Name, Vorname in Druckschrift	Erhaltener Betrag in EUR	Datum	Unterschrift
Gatkowski, Jacek	16,62	26.11.03	[illegible]
Molak, Marcin	22,46	26.11.03	Molak
Kowalczyk, Grzegorz	15,82	26.11.03	G.Kowalczyk
Mikzarek, Grzegorz	19,74	26.11.03	Mik
Kaminska, Kamila	16,16	27.11.03	Kamila Kamińska
Teterycz, Małgorzata	24,88	27.11.03	Małgorzata Teterycz
Zdanowicz, Mariusz	18,96	27.11.03	Mariusz Zdanowicz
Radomski, Wojciech	17,44	27.11.03	Radomski Wojciech
Zielinski, Rafal	15,90	28.11.03	Zieliński
Ziołkowski, Michał	23,42	28.11.03	Ziółkowski Michał
Jasiński, Michał	13,26	28.11.03	M. Jasiński
Bulaszewski, Maciej	14,06	28.11.03	Bułaszewski
Zabdyr, Bartłomiej	13,86	01.12.03	[illegible]
Galas, Jacek	28,90	01.12.03	Galas
Rudzinski, Przemysław	15,06	01.12.03	Rudziński
Siemion, Andrzej	13,78	01.12.03	Siemion
Ludwiczuk, Piotrek	19,68	02.12.03	Ludwiczuk
Slusarczyk, Bogumił	11,60	02.12.03	[illegible]
Wojciech, Kałużny	14,40	02.12.03	Wojciech
Wojtanek, Magda	19,68	02.12.03	Wojtanek

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HOW INDIVIDUALS LEARN TO TAKE TURNS: EMERGENCE OF ALTERNATING COOPERATION IN A CONGESTION GAME AND THE PRISONER'S DILEMMA

DIRK HELBING, MARTIN SCHÖNHOF and HANS-ULRICH STARK

Institute for Transport & Economics, Dresden University of Technology, Andreas-Schubert-Str. 23, 01062 Dresden, Germany

JANUSZ A. HOŁYST

Faculty of Physics and Center of Excellence for Complex Systems Research, Warsaw University of Technology, Koszykowa 75, PL-00-662 Warsaw, Poland

Faculty of Physics and Center of Excellence for
Complex Systems Research (CSR),
Warsaw University of Technology

Conclusions

A spontaneous cooperation can occur in a simple 2x2 game

The cooperation can lead to coherent oscillatory states in players behaviour

Oscillatory states correspond to system equilibrium

A transition to oscillatory states needs spontaneous flipping of players decisions