Atomic Rydberg Units

Our starting point are Gaussian units for our physical equation and amounts. We then *choose*:

$$\hbar = 2m = \frac{e^2}{2} = 1 \tag{1}$$

where m is the mass of an electron and e its charge.

[Remark: We can actually start from SI units too. But since SI units have 4 basic units (m, kg, s and A), (1) is not sufficient to make everything is dimensionless, e.g. $a_0 = 4\pi\varepsilon_0$. The choice $\varepsilon_0 = 1/4\pi$ leads to the same unit for length, energy and time (and this choice is used in the) table. But for now we want to keep things simple.]

So the unit for angular momentum is $\hbar$, the unit for mass is $2m$ and the unit for charge is $e/\sqrt{2}$. Hereby all other units are defined too. The Bohr radius is the unit for length defined as:

$$a_0 \equiv \frac{\hbar^2}{me^2} = 1 \approx 5.29 \cdot 10^{-9} \text{ cm} \tag{2}$$

Likewise the Rydberg energy is the unit for energy:

$$E_{\mathrm{Ryd}} \equiv \frac{e^2}{2a_0} = \frac{me^4}{2\hbar^2} = 1 \approx 2.18 \cdot 10^{-11} \text{ erg} \approx 13.6 \text{ eV}$$

Time is the ratio of an angular momentum and an energy, therefore the unit for time is:

$$t_0 \equiv \frac{\hbar}{E_{\mathrm{Ryd}}} = \frac{2\hbar^3}{me^4} = 1 \approx 4.84 \cdot 10^{-17} \text{ s} \tag{3}$$

Now we know how to convert centimeters, grams and seconds. So it is now straight forward to convert all physical quantities to the CGS system.

The unit of speed is the ratio of length and time, so

$$\frac{a_0}{t_0} = \frac{e^2}{2\hbar} = 1 \approx 1.094 \cdot 10^8 \text{ cm/s} \quad .$$

We can now give a value for the light in vacuum c in atomic Rydberg units:

$$c \approx \frac{2.998 \cdot 10^{10} \text{ cm/s}}{1.094 \cdot 10^8 \text{ cm/s}} \approx 274$$

A more elegant way to get the same result is to use the relation of c with the fine structure constant α:

$$\alpha = \frac{e^2}{\hbar c} = \frac{2}{c} \approx \frac{1}{137} \qquad \longrightarrow \qquad c = \frac{2}{\alpha} \approx 274$$

Gaussian units are good for calculations, but a little cumbersome for giving explicit values for quantities. The table is therefore made for converting quantities in Atomic Rydberg units (numbers), into SI units. The table is arranged so that one Atomic Rydberg unit equals the corresponding number of units in SI.

Conversion Table for Atomic Rydberg and MKSA Units

omic Rydberg units (ARU) defined by: $\hbar = 2m_e = e^2/2 = 1$ (and $4\pi\varepsilon_0 = 1$).

Quantity	Symbol	ARU	MKSA
Length	ℓ	$1{=}a_0$	$5.2917725 \cdot 10^{-11}$ m ≈ 0.5 Å
Mass	m	$1{=}2m_e$	$1.8218779 \cdot 10^{-30}$ kg
Time	t	1	$4.8377687 \cdot 10^{-17}$ s $\approx 1/20$ fs
Frequency	ν	1	$2.0670687 \cdot 10^{16}$ Hz
Speed	v	1	$1.0938457 \cdot 10^{6}$ m/s
Momentum	p	1	$1.9928534 \cdot 10^{-24}$ kg·m/s
Force	F	1	$4.1193647 \cdot 10^{-8}$ N
Power	P	1	45.059494 mW
Energy	E	$1{=}E_{\mathrm{Ryd}}$	$2.1798741 \cdot 10^{-18}$ J ≈ 13.6 eV
Charge	q	$1{=}e/\sqrt{2}$	$1.1329105 \cdot 10^{-19}$ C
Charge density	ρ	1	$7.6452571 \cdot 10^{11}$ C/m^3
Current	I	1	2.3418037 mA
Current density	J	1	$8.3627316 \cdot 10^{17}$ A/m^2
Electric field	E	1	$3.6360903 \cdot 10^{11}$ V/m
Potential	Φ	1	19.241363 V
Polarization	P	1	40.456961 C/m^2
Conductivity	σ	1	$2.2999241 \cdot 10^{6}$ S/m
Resistance	R	1	8.2164712 kΩ
Capacitance	C	1	$5.8878910 \cdot 10^{-21}$ F
Magnetic flux	ϕ	1	$9.3085262 \cdot 10^{-16}$ Wb
Magnetic induction	B	1	$3.3241346 \cdot 10^{5}$ T
Magnetization	M	1	$4.4253673 \cdot 10^{7}$ A/m
Inductance	L	1	$3.9749387 \cdot 10^{-13}$ H
some constants:			
Planck's quantum	$\hbar$	1	$1.05457266 \cdot 10^{-34}$ J·s
$h = 2\pi\hbar$	h	2π	$6.6260755 \cdot 10^{-34}$ J·s
Charge of electron	e	$\sqrt{2}$	$1.60217733 \cdot 10^{-19}$ C
Bohr radius, $\hbar^2/me^2$	a_0	1	$5.29177249 \cdot 10^{-11}$ m
Energy 1 electron Volt	eV	1/13.605698	$1.60217733 \cdot 10^{-19}$ J
Rydberg energy, $e^2/2a_0$	E_{Ryd}	1	$2.1798741 \cdot 10^{-18}$ J
Hartree energy, e^2/a_0	E_{h}	2	$4.3597482 \cdot 10^{-18}$ J
Speed of light	c	$2/\alpha \approx 274$	$2.99792458 \cdot 10^{8}$ m/s
Permeability of vacuum	μ_0	$\pi\alpha^2 \approx 1.7 \cdot 10^{-4}$	$4\pi \cdot 10^{-7}$ H/m
Permittivity of vacuum	ε_0	$1/4\pi \approx 0.08$	$8.854187817 \cdot 10^{-12}$ F/m
Bohr magneton	μ_B	$\sqrt{2}$	$9.2740154 \cdot 10^{-24}$ J/T
Mass of electron	m_e	1/2	$9.1093897 \cdot 10^{-31}$ kg
Mass of proton	m_p	918.07638	$1.6726231 \cdot 10^{-27}$ kg
Mass of neutron	m_n	919.34183	$1.6749286 \cdot 10^{-27}$ kg
Gravitation constant	G	$1.92000 \cdot 10^{-42}$	$6.67259 \cdot 10^{-11}$ N·m^2/kg^2

the fine structure constant: $\alpha = e^2/\hbar c = 1/137.0359895$